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Keywords
Nondestructive Evaluation

Disciplines
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THE IMPACT OF INSPECTION AND ANALYSIS UNCERTAINTY ON RELIABILITY PREDICTION AND LIFE EXTENSION STRATEGY

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ABSTRACT

Life extension of high cost components until measurable damage is detected can result in marked reductions in total cycle costs. The life extension strategy for turbine disks is based upon nondestructive inspection to detect defects, usage and stress analysis to define requirements, and fracture mechanics analysis and testing to evaluate the severity of any defects under future usage. Because there are uncertainties and inaccuracies in the inspection, analysis, testing and definition of past and future usage, the selection of the optimum life extension strategy requires quantitative evaluation of the costs and risks associated with each uncertainty during life extension. This paper summarizes recent developments in the basic methodologies necessary to quantify reliability. Specific examples are described which illustrate the concepts and payoff possible as well as the relative importance of inspection, analysis, and usage uncertainties on the optimum life extension strategy.

INTRODUCTION

Engineering components in high performance equipment may wear out due to fatigue or creep. Historically such components have been designed for replacement after a specified amount of service. The "specific design life" is that beyond which a significant number of failures are predicted to occur by analysis, lab testing and operating experience. Because however, there is significant variability in the loading conditions and the materials response, most nominally identical components could provide reliable service well beyond the "design life", but all components are retired because the precise amount of damage accumulation is not established for each. Figure 1 shows a typical variation in the actual life of nominally identical components. Because so much of the useful life of most components is not utilized, marked reductions in the total cycle costs would result by life extension of individual component until damage actually develops. To achieve such life extension without reducing equipment reliability requires reliable non-destructive inspection to detect defects or damage development, fracture mechanics analysis and testing to evaluate the severity of any defects present under continued operation, and a quantitative method to select the accept/reject conditions for life extension.

This paper will address four topics. First, it reviews briefly the basic concepts associated with probabilistic fracture mechanics (PFM), and retirement for cause (RFC); second, it discusses some recent developments in these methodologies; third, it describes two specific examples which quantify the impact of inspection and analysis uncertainties on the optimum RFC strategy; and fourth, it summarizes progress to date on an ARPA sponsored project to evaluate RFC for application to gas turbine disks.

BASIC CONCEPTS

Fracture Mechanics - The conventional approach to life prediction involves establishing an allowable design life at which all such components are removed from service. The fracture mechanics approach to life prediction differs from the conventional approach in that it acknowledges that defects are present or will develop and that failure will eventually occur by the gradual growth of cracks until they reach a critical size. The fracture mechanics design approach, therefore, establishes design allowables in terms of allowable defect sizes which cannot grow to a dangerous size between inspection intervals.

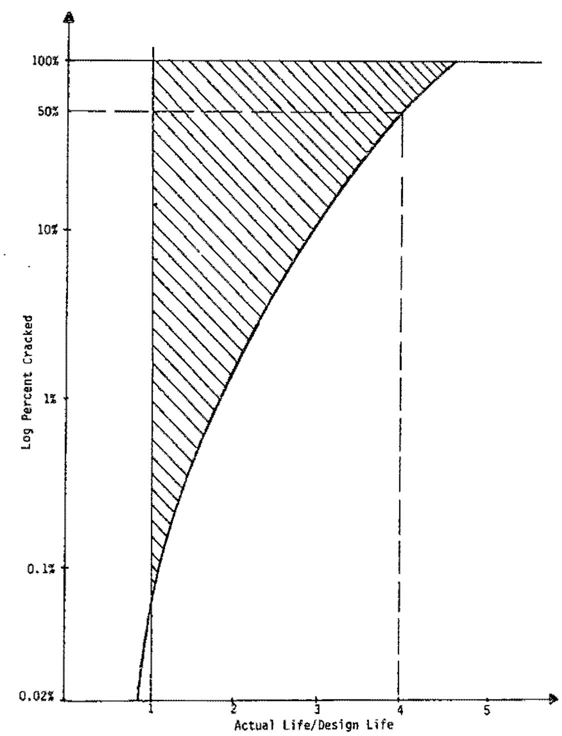

Fig. 1 Schematic representation of the distribution of actual failure lives compared to the nominal design life which is established to assure that less than one in 1,000 components develop a crack.

In order to effectively implement the fracture mechanics approach, Fig. 2 shows the various types of input required. Analysis, materials, and inspection input are all required. As in the conventional design approach, the steady state, thermal, and vibratory stresses must be identified at the critical locations. The crack path through the structure must be calculated and specifically the crack driving force, that is the crack tip stress intensity factor, K, must be evaluated as a function of increasing crack size. Finally, the critical crack size at which unstable fracture occurs or the crack size in which arrest of a growing crack occurs must be calculated. These analytical efforts utilize as input certain materials properties. Specifically the threshold below which high frequency fatigue (HFF) does not occur, the mode of crack propagation, and the materials crack growth law, either for fatigue or creep conditions, must be determined as a function of crack tip stress intensity factor, K, operating temperature, frequency, and other loading conditions. The materials fracture toughness must also be determined and used to evaluate the critical crack sizes. Finally, the inspection input requires definition of the flaw size range of concern, the orientation and shape of flaws of concern and an estimation of the probability that flaws of various size exists prior to inspection of the component. These three types of inputs are combined to perform the lifetime prediction. For example, the number of cycles to grow a fatigue crack to failure (N_F) is calculated by taking the material crack growth law, rearranging and numerically integrating from the initial flaw size (A_i) to the final or critical crack size (A_f) over the appropriate distribution of K as the crack grows.

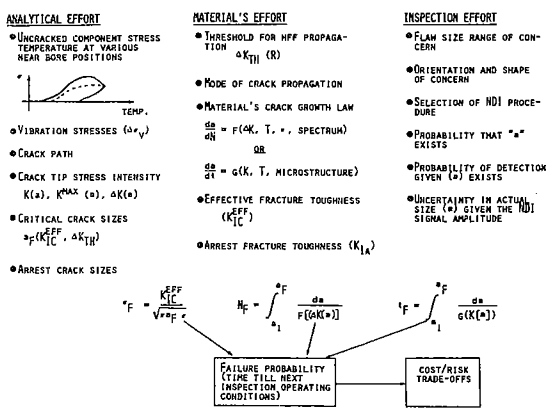

Fig. 2 Probabilistic fracture mechanics approach to lifetime prediction.

Probabilistic Fracture Mechanics - If each of the input parameters are assumed to be known exactly, we get an exact calculation of remaining life time. In practice, however, the input parameters are usually not known exactly; in fact, the uncertainties vary considerably from one input to another. The probabilistic fracture mechanics approach (1,2) accepts uncertainties in the various input parameters, quantifies them, and calculates a failure probability as a function of continued operating time or cycles, rather than a precise remaining lifetime. The engineering community has generally accepted the fact that various input parameters are uncertain and that the probabilistic approach is more realistic than the deterministic life prediction. However, most design engineers do not fully understand the quantitative requirements and do not have the needed tools to actually implement a probabilistic fracture mechanics approach. The combination of this lack of understanding as well as lack of quantitative data on the specific uncertainties involved has limited the full implementation of probabilistic fracture mechanics to a few instances.

Inspection Uncertainty - One of the key concepts which until recently limited the probabilistic analysis was a quantitative understanding of inspection uncertainty (3) and its impact on the reliability of the engineering component. More specifically, the inspector normally establishes an inspection level or sensitivity, shown as b on Fig. 3, and ideally the inspection should locate all imperfections of size greater than S and not indicate the presence of any imperfections smaller than size S. In Fig. 3, this is quantitatively stated as the probability of rejection for various actual flaw sizes (a) at inspection level S [P(R/a, S)]. For an ideal inspection, P(r/a,S) = 1 for crack sizes (a) bigger than S, and P(R/a, b) = 0 for crack sizes a < S. The typical eddy current inspection, like that used to inspect turbine disk bolt holes, is not perfect. As shown in Fig. 3, there is a finite probability of rejecting components with actual imperfections smaller than S, and a finite probability of not rejecting components with actual imperfections larger than S. Two other inspection methods are also shown on the same figure. That labeled A is an inspection method with the same sensitivity as the typical eddy current inspection, that is 50% of the time it rejects imperfections of size S; however, it has a reduced inspection uncertainty and more closely approaches the performance of the ideal inspection which has 0 inspection uncertainty. The dotted line indicates the performance of inspection method A when utilized at a higher sensitivity but with the same inspection uncertainty.

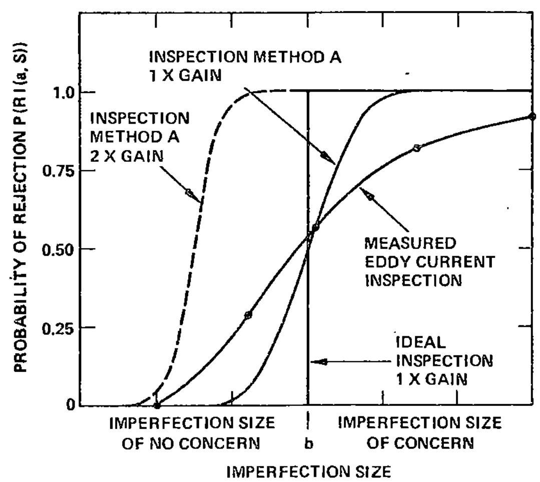

Fig. 3 Comparison of the inspection uncertainty, characterized by the probability of rejection as a function of flaw size, for ideal and real inspections.

Preinspection Material Quality - Qualification of the flaws which may get into service requires quantification of both the inspection uncertainty and the distribution of flaws in the material prior to the inspection (4). Deterministic fracture mechanics approaches have very conservatively assumed that the preinspection flaw frequency is larger for each crack size so that $pn(a) = 1$ and the probability of flaw occurrence in the inspected part was exactly equal to the probability of the inspection missing a flaw of size a, if it exists, which is one minus the probability of rejection, $P(R/a, S)$, given that a flaw of size a exists. More realistically, when the inspection procedures of Fig. 3 are applied to a component which initially contains a distribution $pn(a)$ of imperfections of various size (a), the distribution of imperfection sizes after inspection is modified as shown in Fig. 4. The ideal inspection would eliminate all imperfections of size greater than S, but with real inspections some larger imperfections will get into service and some components with smaller imperfection of no concern will be rejected and thereby increase the total costs. It is this probability distribution of imperfections after inspections, which is the product of the preinspection flaw distribution [pn(a)] and inspection reliability [1 - P(R/a, S)], that would be input as the probable initial flaw size (a_i) for a probabilistic fracture mechanics calculation of failure probability.

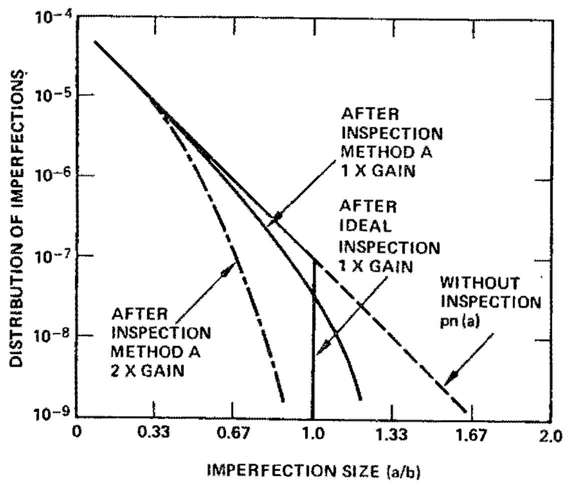

Fig. 4. The effect of various inspections on the distribution of imperfection sizes going into service.

Retirement for Cause (RFC) - A life extension strategy based upon retirement for cause rather than removal at a specified design life, can more completely utilize the available life of each component (5). The RFC procedures may utilize either deterministic or probabilistic fracture mechanics. In the deterministic case, the non-destructive inspection defines the maximum flaw size that could be missed and get into service; loading and stress analyses define the maximum cyclic and steady stresses in the areas of concern; and the deterministic fracture mechanics analysis calculated the maximum amount of crack progression from the largest initial flaw that might occur under continued operation at conservative (highest) cyclic and steady stresses. The RFC allowables are then established by selecting an appropriate safety factor based upon qualitative engineering judgment.

The probabilistic approach to RFC is similar, but instead of a specified maximum flaw size, a probability of occurrence of various flaw sizes is specified. Similarly instead of a maximum cyclic and steady stress, a probability distribution of various cyclic and steady stresses is input, and instead of a specific crack progression curve, the probabilistic fracture mechanics calculation yields a failure probability (reliability) as a function of time. Instead of a specific safety factor an appropriate accept/reject inspection size and inspection interval are selected to maintain a sufficiently low failure probability.

Successful implementation of the probabilistic fracture mechanics approach requires extensive measurements to obtain the statistical data and develop the appropriate probability distributions for the inspection, the mission loads, the local stress concentrations, and the materials crack propagation and fracture toughness properties. Large deficiencies in any one of these input parameters will require utilizing a conservative upper bound and thereby reduce the life extension and payoff which results from implementing the RFC program.

Combined Analysis - Life extension errors, which can reduce the effectiveness of RFC program, can be substantially negated with a modified RFC procedure which makes more direct use of past operating experience as reflected in the inspection information to establish the RFC strategy. This modified approach is a logical outgrowth of a statistical engineering method called Combined Analysis which has been developed by Failure Analysis Associates (5-8). Combined analysis (CA) utilizes the minimum amount of engineering modeling required to supplement the routine statistical analysis of actual in-service data on the frequency and severity of cracking, failures, and successes. Figure 5 shows typical results for a hypothetical but realistic population of turbine rotors. The conventional "design life" is established to assure an acceptably low failure rate. A significant percentage of rotors ($\approx 10\%$) will be cracked but not failed while the balance will not even be cracked at the "design life". An RFC procedure based upon PFM would utilize a conservative calculation of the distribution of crack propagation lives, N_p, to establish allowables. The CA approach would utilize all available in-service data on N_p and the initiation lives, N_i, along with laboratory data or engineering models which relate N_i, N_p and N_f as shown in Fig. 5. As new service or test data reveal past errors in part usage (mission mix) or calculated stresses, the CA procedures continually and intrinsically account for them. Thus errors in the engineering model or materials data are not as critical in a CA/RFC life extension as with PFM/RFC because CA provides continual calibration with actual experience. The basic approach of incorporating a "fudge" factor in the design calculation to explain actual test and field experience is a common design approach. The CA approach simply provides a more formal and mathematically rigorous basis for incorporating actual performance data into the life prediction.

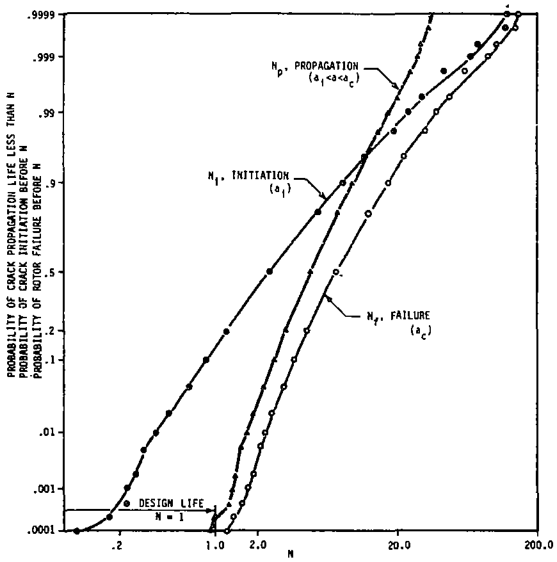

Fig. 5. Comparison of the variability of initiation lives, propagation lives, and total cycles to failure for a hypothetical but realistic population of turbine rotors.

An inspection-based CA/RFC procedure has been proposed which uses actual field data, such as the maximum apparent crack depth (a, measured non-destructively), to calibrate the calculated remaining life. As will be shown, life extension based upon the CA/RFC procedure is much more effective than purely analytical PFM/RFC because it is much less sensitive to analysis errors than PFM/RFC procedures which uses inspection only as a flaw screening device.

Reliability/Cost Optimization - Whether the RFC system is based upon deterministic fracture mechanics, probabilistic fracture mechanics, or combined analysis techniques, selection of the specific life extension strategy should be based upon optimization of the relative cost and reliability associated with various options. Specifically the effects of realistic inspection, analysis, or usage uncertainties must be quantified and incorporated in selection of the optimum life extension strategy.

The following two sections provide examples which illustrate the basis concepts.

TURBINE BLADE: SELECTION OF AN OPTIMUM INSPECTION REJECTION LEVEL

The following quality assurance problem (4) illustrates the use of inspection uncertainty, preinspection flaw frequency, probabilistic fracture mechanics, combined analysis and reliability/cost optimization concepts to select the optimum rejection level for inspections performed during manufacture of a population of gas turbine blades. Figure 6 summarizes the methodology applied in this example. The analysis inputs are: (1) the rejection probability as a function of inspection level (4) and imperfection size (a), (2) the flaw preinspection flaw frequency (FF) as a function of imperfection size, (3) the conditional probability of failure given an imperfection of size a, (4) the manufacturing cost per blade, (5) the inspection cost per blade, and (6) the average cost per failure including the many indirect costs. Four methods have been identified for determining the flaw frequency; and three methods for determining the conditional failure probability, one of which is probabilistic fracture mechanics. In this example, the actual failure history is used to predict the probability of failure given a flaw of size a rather than PFM analysis.

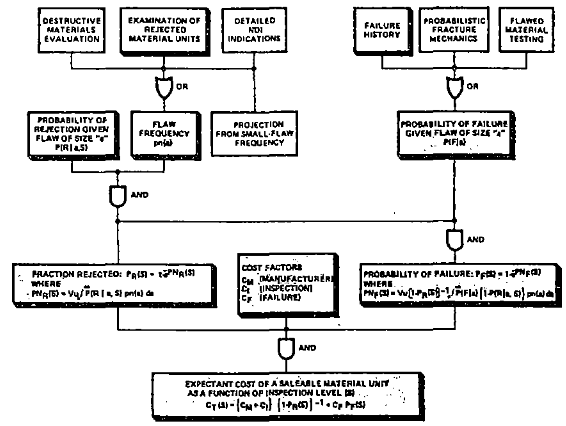

Fig. 6. Combined analysis and inspection procedure to determine the optimum inspection size, which minimizes the total product cost.

First, we consider the failure history. Assume that 100,000 blades have completed their design life, and 100 of the blades failure prematurely. The fraction failed is then $F_F = 10^{-3}$. The total cost of these 100 failures to the manufacturer, including estimated direct costs (e.g., customer relations), is estimated to be 10 million dollars. This given an average cost per failure C_F = $100,000.

The 100 failed blades are analyzed to determine the size of defect which initiated the failure, and the results are summarized in Table 1. A point estimate of the size distribution of defects which historically caused failure P_{no} (F,a,S_0) is given in column three by dividing the number of initiating defects in each size interval by the interval size and the total volume of the material in the 10^5 blades, i.e., by dividing by 50 cm^3/blade x 10^5 blades / 10^{-1} cm = 5 x 10^5 cm^4.

The turbine blades before being admitted to service had to pass the historical inspection in which the inspection uncertainty was $\delta = .2S + 0.1$ cm with the inspection size $S = S_0 = 3/4$ cm, and the rejection rate has been $F_{R0} = 4.5\%$. A sample of 100 rejected blades were examined, and the imperfections in these blades which cause rejectable indications are summarized in Table 2. A point estimate of pn_0 $((a,S_0)|R)$, the size distribution of imperfections which cause rejection, given that the blade has been rejected, is given in column three of Table 2. It is obtained by dividing the number of rejectable indications in each imperfection size interval by the volume of the 100 blades and the interval size, i.e., column two is divided by 100 blades x 50 cm^3/blade x 10^{-1} cm = 500 cm^4.

Table 1. Hypothetical problem: Number of failure initiating flaws of various sizes in 100 failed blades.

Flaw Size Interval in cm	Number of Flaws	$pn_o(F,a,S_o)$ in cm^{-4}	Flaw Size "a" in cm
0.00 - 0.10	0	0×10^{-6}	0.05
0.10 - 0.20	0	0×10^{-6}	0.15
0.20 - 0.30	0	0×10^{-6}	0.25
0.30 - 0.40	0	0×10^{-6}	0.35
0.40 - 0.50	1	2×10^{-6}	0.45
0.50 - 0.60	7	14×10^{-6}	0.55
0.60 - 0.70	25	50×10^{-6}	0.65
0.70 - 0.80	36	72×10^{-6}	0.75
0.80 - 0.90	22	44×10^{-6}	0.85
0.90 - 1.00	7	14×10^{-6}	0.95
1.00 - 1.10	2	4×10^{-6}	1.05
1.10 - 1.20	0	0×10^{-6}	1.15

Table 2. Hypothetical problem: Number of rejectable indications in each flaw size range in 100 rejected blades.

Flaw Size Interval in cm	Number of Rejectable Indications	$pn_o((a,S_o),R)$ in cm^{-4}	Flaw Size "a" in cm
0.00 - 1.10	16	32×10^{-3}	0.05
0.10 - 0.20	19	38×10^{-3}	0.15
0.20 - 0.30	19	38×10^{-3}	0.25
0.30 - 0.40	17	34×10^{-3}	0.35
0.40 - 0.50	13	26×10^{-3}	0.45
0.50 - 0.60	9	18×10^{-3}	0.55
0.60 - 0.70	5	10×10^{-3}	0.65
0.70 - 0.80	3	6×10^{-3}	0.75
0.80 - 0.90	1	2×10^{-3}	0.85
0.90 - 1.00	1	2×10^{-3}	0.95

Manufacturing and Inspection Costs - If the cost of manufacturing a blade is \$100 plus an additional \$10 to inspect the blade, the question is whether the total cost could be reduced by selecting a different inspection level (S). Using the data above as input, the dependence of the expectant cost per turbine blade upon inspection level (S) can be determined as indicated in Fig. 7. First the probable fraction of rejected blades is determined as a function of S. Assuming that the rejection probability for the historical inspection method has been determined to be

$$P_o(R|a,S) = (\delta_o(S)\sqrt{2\pi})^{-1} \int_{-\infty}^{a} \exp\left[-(X-S)^2/2\delta_o^2(S)\right] dx \quad (1)$$

with the historical inspection method uncertainty given by

$$\delta_o(S) = 0.2S + 0.1 \text{ cm} \quad (2)$$

the rejection probability for the specific inspection used on the 100,000 turbine blades (the historical inspection method with S = 0.75) is given by

$$P_o(R|(a,S_o)) = (\delta_o(S_o)\sqrt{2\pi})^{-1} \int_{-\infty}^{a} \exp\left[-(X-S_o)^2/2\delta_o^2(S_o)\right] dx. \quad (3)$$

where S_o = 0.75 cm. The probable fraction of rejected blades if the historical inspection method is used is given by

$$F_{RO}(S) \simeq 1-e^{-PN_{RO}(S)} \quad (4)$$

where

$$PN_{RO}(S) = V_u F_{RO}(S_o) \int_0^{\infty} \left[P_o(R|(a,S))\right]\left[P_o(R|(a,S_o))\right]^{-1} pn_o((a,S_o)|R)\, da. \quad (5)$$

$F_{RO}(S_o) = 0.045$, $V_u = 50\ cm^3$, and $pn((a,S_o)|R)$ is given in Table 2.

The average cost to manufacture a turbine blade which passes the historical inspection method as a function of inspection level (S) is then given by

$$(C_M + C_I)(1-F_{RO}(S))^{-1} \quad (6)$$

and is illustrated in Fig. 7. Here $C_M + C_I$ = \$110.

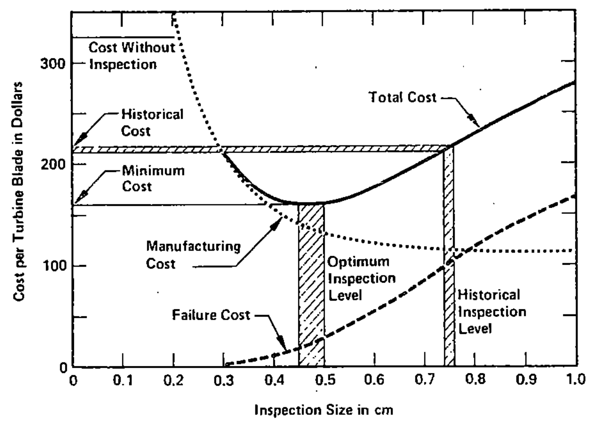

Fig. 7. Total cost per turbine blade as a function of rejection level when the average cost of a failure is \$100,000.

Failure Costs - Next the probable fraction of blades which would fail is given by

$$F_{FO}(S) \simeq 1-e^{-PN_{FO}(S)} \quad (7)$$

where

$$PN_{FO}(S) = V_u \frac{1-F_{RO}(S_o)}{1-F_{RO}(S)} \int_0^{\infty} \frac{1-P_o(R|(a,S))}{1-P_o(R|(a,S_o))}\, pn_o(F,a,S_o)\, da. \quad (8)$$

Here $pn_o(F,a,S_o)$ is given in Table 1, $F_{RO}(S)$ is calculated by Eq. 4, $P_o(R|(a,S_o))$ is given by Eq. 3, $P_o(R|(a,S))$ is given by Eq. 1, $V_u = 50\ cm^3$, and $F_{RO}(S_o) = 0.045$. The probable failure cost per blade in service as a function of inspection size is given by

$$C_F F_{FO}(S) \quad (9)$$

where C_F = \$100,000. This probable failure cost per blade is also shown in Fig. 7.

Inspection Level Optimization - The total expectant cost of a saleable turbine blade is the sum of average cost to manufacture a turbine blade which passes the inspection and the expectant cost due to the finite probability that the blade will fail. The total expectant cost per saleable blade is also illustrated in Fig. 7. The total expectant cost of a turbine blade if no inspection is conducted was calculated to be \$328. It is evident from Fig. 7 that the historical inspection level set by engineering judgment reduces the total cost of a saleable blade to \$215. However, this analysis shows that the total expectant cost of a blade can be further reduced from the present cost of \$215 per blade to \$159 per blade by reducing the inspection size from the historical level of S = 0.75 cm to S = 0.45 cm. Over the 10^5 blades, this represents a potential additional savings of approximately 6 million dollars by simply adjusting the rejection level for an existing inspection.

Now consider the optimum decision from the user rather than manufacturer's point of view. The user might experience an additional loss on the average of \$900,000, which results from the fact that a blade failure forces the turbine out of service for an extended period of time. Hence the average failure cost to the user might be \$1,000,000. Figure 8 illustrates the results of a similar optimization analysis where the new expectant costs of a saleable blade is very high (\$1,115 per blade) if the inspection size is left at the historical level. A change in inspection size from the historical level of 0.75 cm to the level of 0.35 cm will reduce the total expectant cost to the user of each blade to \$234.

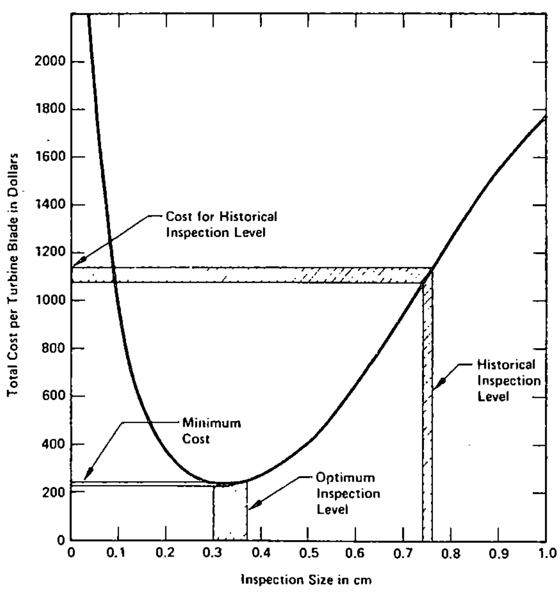

Fig. 8. Total cost per turbine blade as a function of inspection rejection level when the average cost of a failure is a million dollars.

Alternative inspection procedures with modified uncertainty have been (5) similarly evaluated to determine (1) the specific inspection level which produces the minimum cost, and (2) the magnitude of the minimum cost relative to the historical blade inspection utilized at the level which produces minimum cost. The quantitative comparisons clearly show the importance of selecting the optimum rejection level for the specific inspection and the importance of low inspection uncertainty, rather than high resolution alone, in minimizing the total cycle costs.

TURBINE DISK: IMPACT OF ANALYSIS AND INSPECTION UNCERTAINTY ON LIFE EXTENSION STRATEGY

FAA has developed analytical procedures (5-9) to evaluate the effectiveness of any proposed life extension procedure based upon retirement for cause (RFC), considering the in-service loading and analysis uncertainties as well as inspection uncertainty. This evaluation procedure, a probabilistic simulation model, has been applied to a population of 10,000 gas turbine disks to determine the impact of

a) Stress variations from disk to disk

b) Unknown precise age of past usage of the disks

c) Multiple (repeated) inspections rather than a single inspection

d) Reduced inspection intervals, and

e) Utilizing inspection results, through combined analysis (CA), rather than design calculations to establish RFC allowables.

The effect of analysis and inspection uncertainty was evaluted by considering the six hypothetical teams shown in Fig. 9. Each team consists of one analyst and one inspector of varying capabilities, and the total expectant cost savings possible through RFC procedures utilizing specific teams were computed for 1000 disks, averaged, and compared.

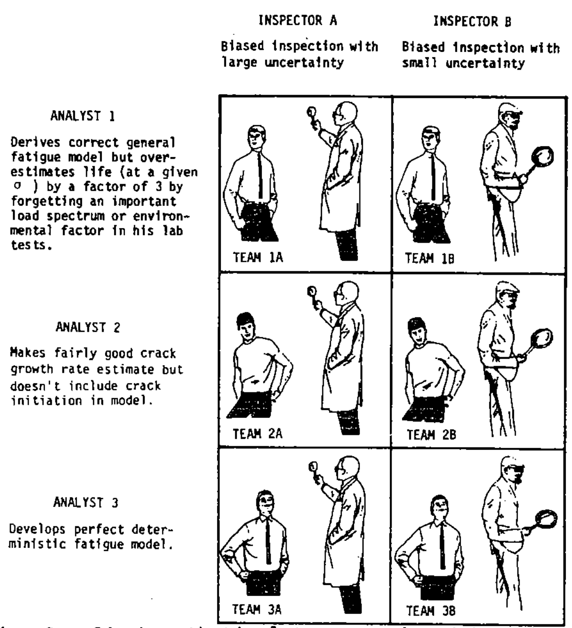

Fig. 9. Six hypothetical teams assigned to set a retirement-for-cause-based life limit for 10,000 inspected turbine disks.

Fatigue Life Simulation - The total fatigue life to brittle failure of a disk when a life-limiting rim can be expressed generally as

$$N_f = N_i + N_p$$

where

N_f = Life to failure of the shortest-lived of all the (up to 100 or more) rim slots of the rotor (in units of cycles, time, inspection intervals, or design lives)

N_i = Life to initiation of a crack of some small defined depth, a_i, (a_i = 0.001 inches in this report)

N_p = Life to propagate the crack from size a_i to critical size for brittle failure, a_c.

The specific equation used to simulate in-service fatigue life required for failure is given by

$$N_f = C_i/\sigma^6 + C_p \log (a_c/a_i)/\sigma^3. \qquad (10)$$

The equation used to relate the crack depth a to the number of applied load cycles N is

$$N = C_i/\sigma^6 + C_p \log (a/a_i)/\sigma^3 \qquad (11)$$

where

a = Crack depth (greater than a)

N = Number of vendor-specified design lives required to produce a crack of depth a

N_f = Dimensionless life expressed as the number of "worst-case" predicted retirement lives required for failure (i.e., a rotor with N_f = 7 would fail after seven times the predicted life)

σ = Effective alternative* nominal stress at the crack locus in ksi (treated as a random variable)

C_i, C_p = Parameters (treated as random variables) which simulate the variation of crack initiation and crack growth, respectively, at a given σ, as caused by geometric variables such as surface roughness and metallurgical variables such as local hardness or composition, and

$$a_c = (K_c/(2.5\sigma))^3 \qquad (12)$$

with

K_c = Critical stress intensity factor expressed in ksi $(\text{in.})^{1/2}$ (treated as a random variable).

The form of (10) and (11) and the numerical values of the parameters have been selected for simplicity and because they are representative of observed fatigue performance of certain gas and steam turbine rotor rims.

The exponent 3 in Eq. 12 accounts for the stress decrease below the crack surface (an exponent of 2 would be correct for certain crack geometries subject to uniform stress), and the factor 2.5 (with inherent units of in.$^{1/6}$* to make Eq. 3 dimensionally correct) is representative of the results of a stress intensity factor analysis of the rim crack geometry.

The cumulative probability distribution of C_i is assumed to be log-normal so that log C_i is a Gaussian or normally distributed variable. The selected parameters of this normal distribution are a mean or median of 10 and a standard deviation of 0.2. These assumptions about the probability distribution of log C_i can be compacted into the notation

$$\log C_i = \text{GAU}\ (10, 0.2) \qquad (13)$$

or

$$C_i = 10^{\text{GAU}(10,\ 0.2)} \text{ksi}^{6**} \qquad (14)$$

The other assumed probability distributions are given by

$$C_p = 10^{\text{GAU}(5.0,\ 0.1)} \text{ksi}^3 \qquad (15)$$

$$\sigma = \text{GAU}\ (40 \text{ ksi},\ 5 \text{ ksi}) \qquad (16)$$

$$K_c = \text{GAU}\ (100 \text{ ksi (in)}^{1/2},\ 15 \text{ ksi (in)}^{1/2})^{***} \qquad (17)$$

Figure 10 shows the probability distribution for inspectors A and B and Fig. 11 that for the operating stress (σ) given by (16). Considerable rotor-to-rotor stress variation has been assumed.**** This stress variation might result from aircraft mission differences which produce different in-service, thermomechanical transients.

Equations 10 through 17 and the numerical values of their parameters have been chosen to simulate many characteristics of real in-service fatigue performance. These include: a greater effect of stress on initiation life (exponent of 6) than on crack growth life (exponent of 3), greater scatter in initiation than in crack growth at a given σ (Eqs. 14 and 15), and a realistic, smaller increase in crack growth rate with crack depth for a crack in a stress concentration than for one under uniform stress.

*Steady stress effects are assumed to be negligible in this example. Refer to (4) for an RFC procedure that considers the relative contribution of steady and alternating sources.

*1 in. = .0254 m

**1 ksi = 6.894×10^6 Pa = 6.894 N/mm^2

***1 ksi $(\text{in})^{1/2}$ = 34.745 Nmm$^{-3/2}$

****The coefficient of variation V_σ for the stress distribution is equal to 5/40 or 12.5%.

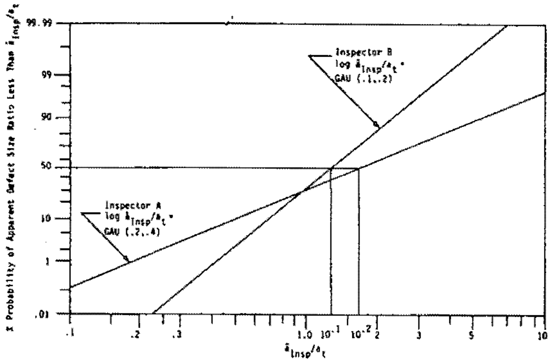

Fig. 10. Probability distribution of $\hat{a}_{Insp}/a_t$ for Inspectors A and B.

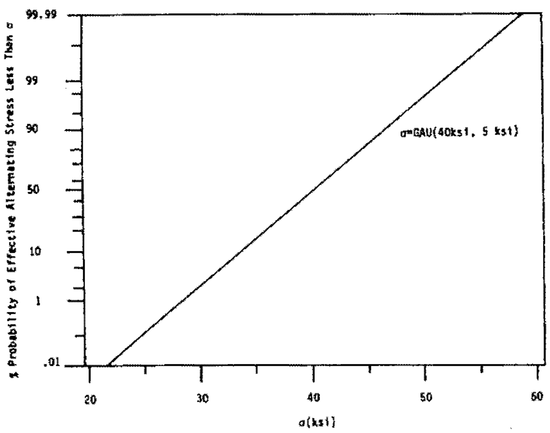

Fig. 11. Probability distribution of σ due to rotor-to-rotor variation.

Figure 5 presents the simulated life-to-failure N_f (also N_i and N_p) data for 10,000 rotors* obtained by applying the Monte Carlo simulation computer program in (2) to Eqs. 10 through 17. The results show that crack propagation, N_p, controls the early part of the N_f probability distribution and crack initiation the later part ($N > 20$). Furthermore, significant fatigue life scatter occurs and the first few failures occur between N = 1, the design life, and N = 1.5. This is intended to simulate a good initial life prediction or design analysis resulting in neither failure nor severe overdesign.

Actual Crack Depth at Inspection Time N_t - The actual crack size a_t at the actual inspection time N_t is solved from (11) as

$$\log a_t = (N_t - C_i/\sigma^6)(\sigma^3/C_p) + \log a_i \quad (18)$$

For further discussion, it is of interest to note that at the median values ($N_t = 1$, $C_i = 10^{10}$, and $C_p = 10^5$), a 7% to 8% change in σ will change a_t by a factor of approximately three. Thus, for each in-service rotor, an inspection error of a factor of three in the estimate of crack depth is equivalent to a 7% to 8% analysis error in the effective nominal alternating stress.

*These results, from an earlier 10,000 rotor simulation (1), also accurately represent the results of the present 1000 rotor simulation.

Apparent Crack Depth from Inspection N_t - The inspection uncertainty is simulated by calculating apparent crack depth from

$$\hat{a} = \text{maximum } (.001", \hat{a}_{Insp}) \quad (19)$$

where $\hat{a}_{Insp}$ is derived solely from the calibrated inspection signal given by

$$\log (\hat{a}_{Insp}/a\ ) = \text{GAU } (b,u) \quad (20)$$

where b, taken to be either 0.1 or 0.2, reflects positive bias in the inspection (for example, b = .1 implies that typically the crack size will be overestimated by a factor of $10^{\cdot 1} = 1.26$). This bias could reflect conservative procedures or the fact that multiple rim slots create more change for a high, rather than low, estimate of the rotor's maximum value of a_t. Finally, u, taken to be either 0.2 or 0.4, is the "logarithmic standard deviation" of $\hat{a}_{Insp}/a_t$ and reflects inspection uncertainty in a similar manner as in (12). Eq. 20 is plotted in Fig. 10 for the two sets of parameters b and u, corresponding to inspectors A and B. Inspector A has available a relatively poor technique with large bias and uncertainty, while Inspector B has less bias and uncertainty.

Analysis Uncertainty - Three hypothetical analysts are considered. Analyst 1 uses the equation

$$\hat{N}_f = 3 \times 10^{10}/\hat{\sigma}^6 + 10^5 \log (\hat{a}_c/.001)/\hat{\sigma}^3 \quad (21)$$

to model the fatigue process. By comparing Eq. 21 to Eqs. 10,11, and 23, one can see that the form of Eq. 21 is correct, but that, on average, Eq. 21 will overpredict the median failure life by a factor of three. Such an error could be due, for example, to the use of inappropriate temperatures for laboratory fatigue tests.

Analyst 2 uses the equation

$$\hat{N}_f = .333 \times 10^{10}/\hat{\sigma}^6 + .333 \times 10^5 \log (\hat{a}_c/.001)/\hat{\sigma}^3 \quad (22)$$

to model the fatigue process. On the average, this analyst underpredicts life by a factor of three.

The third analyst develops a near-perfect deterministic model of the fatigue process that corresponds closely to the median life. Analyst 3's equation is

$$\hat{N}_f = 10^{10}/\hat{\sigma}^6 + 10^5 \log (\hat{a}_c/.001)/\hat{\sigma}^3 \quad (23)$$

It has been assumed that the analysts have included all relevant failure modes in their assessments. For example, the effect of a larger-than-anticipated vibratory stress could cause the effective critical crack depth to be limited by high frequency fatigue threshold rather than the material's fracture toughness. The turbine history, destructive metallographic examination of rim slots, and rotor-inspection data can be used to determine if vibratory stresses affect a_c and N_f significantly.

Cost Analysis - The RFC procedures described above have been programmed into a Monte Carlo simulation program which simulated 1000 individual rotors for each RFC procedure and team. The program does the following for each rotor: (1) generates "in-service" fatigue data, (2) performs a chosen RFC procedure on each rotor at the appropriate time and makes random errors using the probability distribution input and other appropriate equations, and (3) checks for failure of the rotor. Costs are assigned to the various outcomes of the RFC procedure for the jth rotor. Each time the rotor is inspected, a negative dollar gain (cost) of

$$G_{ji} = -2000 \text{ dollars}$$

is assigned. Each time the life of the rotor is extended, a gain of

$$G_{je} = 20{,}000\ \hat{N}'_e \text{ dollars}$$

is assigned, where \$20,000 is the original cost of a rotor designed for one life unit and $\hat{N}'_e$ is the perceived amount of life extension until either the next inspection, retirement, or failure, whichever occurs. Should a failure occur before the rotor is retired, a negative gain (cost) of

$$G_{jf} = -1{,}500{,}000 \text{ dollars}$$

is assigned.

Clearly, the estimation of the expected cost of failure G_f is a complex, controversial subject (6,9,10,11) that touches on a variety of sensitive safety, economic, and political issues. However, G_f is finite, and the failure probability is greater than zero and to insist otherwise is unrealistic and impractical. If specification of G_f is undesirable, a maximum allowable failure probability can be specified instead of built into the RFC procedure constraints. This failure probability could be specified and justified by using several comparative criteria. For example, the failure probability may be acceptable if it is less than the in-service failure probabilities demonstrated during the design life of similar equipment accepted by society for general usage. For further discussion of the "how safe is safe enough" question, the reader is referred to the work of Starr (10) and Tetelman (11).

The total RFC cost savings for each rotor is obtained by summing

$$G_j = f_i G_{ji} + f_e G_{je} + f_f G_{jf}$$

(Repeated indices do not denote summation)

where f_i, f_e, f_f represent the number of incidents for each type of cost gain for the jth rotor.

The expected average dollar gain per rotor of the RFC procedure is then estimated by averaging all the simulations

$$\overline{G} = \sum_{j=1}^{1000} G_j/1000.$$

$\overline{G}$ is a measure of the RFC performance. The rms error of the $\overline{G}$ estimate (i.e., the sampling tolerance or standard deviation of $\overline{G}$) due to the use of the finite number of rotor simulations (1000) is estimated to be \$2000 near the optimum safety factor, where the simulated failure probability is of the order of .001. Thus, the curves reflect the simulated procedure with an accuracy of approximately ±2000 dollars.

RFC Evaluation Results - The average dollar gain (cost savings) G for each RFC procedure and analyst/inspector team was computed. Typical results are summarized by Figs. 12-14 and discussed below.

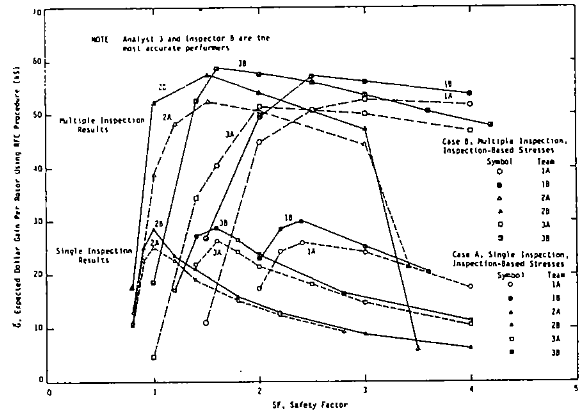

Fig. 12. Expected dollar gain per rotor for single and multiple inspection RFC procedures with various analysis and inspection uncertainties.

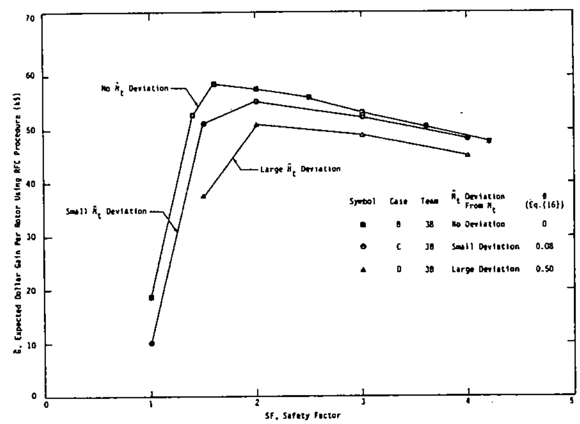

Fig. 13. Effect of uncertain past usage on expected dollar gain per rotor for multiple inspection RFC procedure.

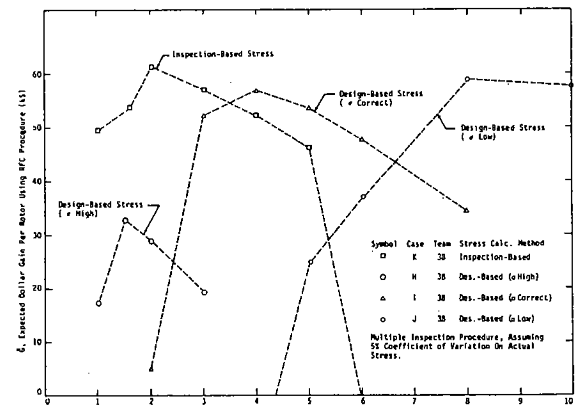

Fig. 14. Comparison of the expected cost savings per rotor with RFC procedures utilizing various stress input.

Single Inspection vs. Multiple Inspection - The $\bar{G}$ results for Cases A and B, single and multiple inspection-based-stress RFC procedures, respectively, are displayed in Fig. 12. Three conclusions are evident: (1) an optimum safety factor exists for each team within each case, (2) the teams with the better inspectors do consistently better than their less able counterparts although even the less accurate inspectors can still achieve substantial cost savings, and (3) provided that the safety factor is between 2 and 3, the multiple inspection RFC procedure is substantially better than the single inspection procedure.

The general shape of the $\bar{G}$ curve is due to the trade-off between premature failures and premature retirement. The optimum safety factor represents the best balance between these two competing effects and corresponds to simulated failure rates of the order of one failure in 1000 rotors. The sharp drop in the $\bar{G}$ curves on the low SF end corresponds to too many failures. The gradual drop in the $\bar{G}$ curves on the high SF end represents the cost of an increasing number of premature rotor retirements.

Figure 12 shows that the optimum safety factor is, unfortunately, a strong function of the analytical model. Since we assume that little or no knowledge of analysis error is available prior to formulation of the RFC procedure, there seems little chance to choose accurately an optimum safety factor. The situation would be significantly worse, however, if the minimum inspection interval was 1 rather than 1/2 times the design life as used. This computation showed an abrupt decrease in $\bar{G}$ at high SF values due to excessive premature retirements caused be a minimum inspection interval that is too large. The results showed that reduced inspection interval reduces the sensitivity to SF, as long as SF is chosen large enough to prevent failures. Furthermore, if an appropriately small minimum inspection interval is used, safety factors between 2.0 and 3.0 produce very substantial economic gains for all teams using the multiple inspection procedure.

It should be noted that none of the RFC procedures studied thus far have taken into account information received from either other rotors or earlier inspections of the same rotor. Thus, neither the analyst nor the inspector is allowed to learn from past experience. This simplification in the model is particularly unfair to the multiple inspection inspector who is made to ignore the results of previous inspections each time the rotor is reinspected.

Known vs. Estimated Component Age and Inspection Interval - It is quite likely that an analyst will not know exactly how many design lives have actually been used at the time of inspection. Figure 13 shows the relatively small effect of this uncertainty on the $\bar{G}$ results of the multiple inspection, CA/RFC procedure applied by Team 3B. The three usage uncertainties evaluated were:

1) Exact knowledge of N_t ($\beta = 0$)

2) A small deviation of N_t from $\hat{N}_t$ (factor of 1.2 ($\beta = 0.8$), i.e., 20% error or less, 68% of the time)

3) A large deviation of N_t from $\hat{N}_t$ (factor of 2 error or less ($\beta = 0.50$), 68% of the time)

Design-Based-Stress (PFM) vs. Inspection-Based Stress (CA)/RFC Procedures - The results of using a single best estimate design stress in Eq. 11 rather than a stress calculated from inspection information is shown in Fig. 14. The results in Fig. 14 are for a rotor population with realistic actual stress variations from rotor to rotor (a larger variation has been assumed throughout the rest of this study). The figure shows that the design-based procedure does reasonably well provided that the analyst chooses the correct stress value. Note, however, that if too high or too low a stress value is chosen, very poor $\bar{G}$ results are obtained, even with Team 3B, the best team. Thus, the inspection-based-stress RFC procedure is much more likely to produce substantial economic gains than the design-based procedure.

SUMMARY AND CONCLUSIONS

The results of this study have led to the following general conclusions:

1) It is evident that even with large inspection and analysis uncertainties, cost effective rotor life extension at extremely low failure probabilities can be effected using a Retirement-For-Cause (RFC) procedure which makes full use of in-service structural fatigue data.

2) Any proposed RFC procedure should be subjected to a parametric probabilistic evaluation using realistic simulated data to evaluate the procedure and to learn which areas of analysis, experiment, logistics, and inspection are most critical to the success of the RFC procedure.

3) Computer simulation of the fatigue crack initiation and growth process provides a viable means for evaluating the effect of both systematic and stochastic errors upon the payoff potential of an RFC procedure.

4) A more effective CA/RFC procedure would be based on probabilistic rather than deterministic life extension calculations and would make use of information obtained from other rotors and previous inspections of the same rotor.

A number of conclusions have been reached for the specific RFC example described. They are:

1) Life based safety factors of 2 to 3 will result in substantial economic gains with little chance of producing an unacceptably large number of failures if the CA/RFC procedure uses a stress value which is calculated from the inspection results, and if the minimum inspection interval is sufficiently small.

2) With regard to subcritical crack growth life, knowledge of the maximum crack depth to within a factor of three is equivalent to knowledge of the effective stress to within 8%.

3) Larger economic gain results from an RFC procedure which uses stress values calculated from inspection results rather than

conventionally calculated stress values.

4) Multiple inspection RFC procedures are at least twice as effective as single inspection procedures if a safety factor of 2 to 3 is used.

5) Uncertainty regarding the age at inspection has little effect on the overall results of the RFC procedure, per se, but may lead to early failures before the first scheduled RFC inspection.

6) Shortening the length of the minimum inspection interval in multiple inspection procedures can substantially reduce the overall cost due to premature retirements.

RECOMMENDATIONS AND ONGOING WORK

In addition to the results described in this paper, Failure Analysis Associates has also evaluated combined analysis/retirement for cause (CA/RFC) procedures for more difficult situations (5,8) where (1) the engineering model of the failure includes systematic and severe errors, like incorrect assessment of the failure mode, or (2) a key life controlling parameter is unavailable, e.g., the part is uninspectable. These studies have shown that cost effective life extension can be achieved using CA/RFC techniques even with serious analysis and inspection uncertainty. Furthermore, any proposed RFC procedure should be evaluated through parametric probabilistic simulation of the service population. These sensitivity studies should identify those analysis, inspection, logistics, and service data which are most critical to optimum payoff of the RFC procedure and the quality of the data required to assure sufficient payoff to implement.

Failure Asalysis Associates has initiated a three year ARPA-sponsored program with the Air Force (AFML) to develop a quantitative methodology and apply it to predict optimum RFC strategy for engine disk life extension. This program is now evaluating experimentally the inspection uncertainty of four disk bolt hold inspection systems using (1) conventional eddy current equipment, (2) conventional eddy current equipment with adaptive learning signal processing, (3) a higher resolution controlled reluctance eddy current (CREC) probe, and (4) a CREC probe with adaptive learning signal processing. The program will estimate the preinspection flaw sizes from the inspection results and calculate using both the probabilistic fracture mechanics and combined analysis approaches, the conditional failure probability for continued operation. A cost equation will be developed and used to assess the payoff of various RFC strategies. The methodology will be verified by a structural simulation testing of a number of bolt hold specimens subjected to the optimum RFC inspection and life extension procedure.

Additional work will be required to quantify the analysis, inspection and materials uncertainty information for the range of components and parts where RFC can produce cost savings.

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